



# **LANGUAGE & LOGIC**

## LANGUAGE & LOGIC

### GLOSSARY

**argument** A collection of premises offered in support of a conclusion. For example:  
(1) All men are mortal; (2) Socrates is a man;  
(3) Therefore, Socrates is mortal.

**conclusion** The statement that an argument attempts to prove. In the argument '(1) All men are mortal; (2) Socrates is a man; (3) Therefore, Socrates is mortal', (3) is the conclusion.

**deduction** An inference from a general claim to a particular conclusion. For example: all snails eat lettuce, this thing is a snail, therefore this thing eats lettuce.

**definite description** An expression that picks out a person, place or thing, for example, 'The last man standing'.

**induction** An inference from many particular claims to a general claim, or to other particular claims. For example: this snail eats lettuce, this snail eats lettuce, this one too, etc, therefore, all snails eat lettuce.

**inference** A mental movement from premises to a conclusion. Sometimes also used as a synonym for 'argument'.

**logic** The study of inference. Logic itself has many branches and manifestations, from informal logic (which examines the structure of argumentation in natural languages) to formal logic (the study of the purely abstract, formal structure of inference), to the study of such things as mathematical reasoning, modality, computer science, fallacies, probability and much else.

**logical form** This is revealed through an analysis of the hidden logical structure underlying the superficial syntax of propositions, according to some philosophers. Bertrand Russell, for example, argued that one can get around certain problems associated with referring to something that does not exist by unpacking the hidden logical form of certain suspect expressions.

**paradox** This involves a certain sort of tension between two claims that seem obviously true. The trouble often comes when conflicting claims seem to follow logically from something else thought to be true.

**predicate** The part of a proposition that attributes something to the subject. That which is stated or asserted about the subject. For example, in the proposition, 'Socrates is drunk', 'drunk' is the predicate.



**premise** A statement advanced in support of a conclusion. In the argument '(1) All men are mortal; (2) Socrates is a man; (3) Therefore, Socrates is mortal', (1) and (2) are premises.

**reference** The object referred to by an expression, according to some philosophers of language, and logicians. For example, the reference of 'Mark Twain' is the actual person, Mark Twain.

**sense** The cognitive significance of an expression, or the way in which something is expressed, according to some philosophers of language, and logicians. For example, the expressions 'Mark Twain' and 'Samuel Clemens' refer to just the same thing, exactly one person. The difference between the expressions, then, has to do with their different senses.

**subject** The part of a proposition about which something is attributed. For example, in the proposition, 'Socrates is drunk', 'Socrates' is the subject.

**validity** The way premises and conclusions hang together logically in successful arguments. If the premises are true and the argument is valid, then the conclusion has to be true.

# ARISTOTLE'S SYLLOGISMS

the 30-second philosophy

More than 2,300 years ago, Aristotle noticed that in certain inferences it is impossible for their premises to be true and their conclusions false. An example is the inference from 'All men are mortals' and 'All mortals fear death' to 'All men fear death'. In modern logic, such inferences are said to be deductively valid. Aristotle discovered that the validity of an inference depends not on its subject matter, but only on the form of the premises and conclusion. All inferences of the form 'All Fs are Gs, and All Gs are Hs, therefore All Fs are Hs' are valid. He described a number of such forms, which are called 'syllogisms'. Until the nineteenth century, the subject of logic pretty much consisted of Aristotle's syllogisms. But syllogisms are only a small portion of all valid inferences, and do not include many of the patterns of valid inference that are employed in science and mathematics. In 1879 Gottlob Frege devised a much more general characterization of valid inference that is sufficient for representing mathematical and scientific reasoning. A descendant of Frege's system, called 'First Order Logic with Identity', is now generally thought to be capable of representing mathematical theories and proofs, and is taught to all philosophy students.

## 3-SECOND THRASH

An inference (or argument) is valid when it is impossible for its premises to be true and its conclusion false.

## 3-MINUTE THOUGHT

In the twentieth century two great mathematical results were proved concerning first order logic: it is complete, and it is undecidable. Kurt Gödel demonstrated that it is possible to programme a computer to list all the valid inferences (completeness), and Alonzo Church demonstrated that it is impossible to programme a computer to determine whether or not every inference is valid (undecidability).

## RELATED PHILOSOPHIES

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RUSSELL'S PARADOX & FREGE'S LOGICISM  
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## 3-SECOND BIOGRAPHIES

ARISTOTLE  
384-322 BCE  
GOTTLLOB FREGE  
1848-1925  
KURT GÖDEL  
1906-78  
ALONZO CHURCH  
1903-95

## 30-SECOND TEXT

Barry Loewer

*To Aristotle it was logical – we are people, we are going to die, and therefore we are frightened. Thanks a lot, Aristotle.*

# RUSSELL'S PARADOX & FREGE'S LOGICISM

the 30-second philosophy

## 3-SECOND THRASH

The collection of all collections that are not members of themselves is itself both a member of itself and not a member of itself.

## 3-MINUTE THOUGHT

Here is a paradox involving reasoning similar to Russell's: 'There is a barber who shaves all those and only those who don't shave themselves'. If the barber shaves himself then he doesn't shave himself, and if he doesn't then he does. This paradox is easy to solve, simply by accepting that there cannot be such a barber. Frege couldn't accept the analogous way out for collections, since he used his principle to prove the existence of collections required by mathematics.

Bertrand Russell thought up a deep and perplexing paradox when reading about Gottlob Frege's system of logic. Frege thought that he could define all mathematical concepts and prove all mathematical truths solely from principles of logic. The view that mathematics can be reduced to logic in this way is called logicism. Had Frege demonstrated the truth of logicism, it would have been one of the greatest achievements in the history of philosophy. But his version of logicism was not successful. One of the logical principles used to prove the existence of numbers, functions and other mathematical objects is: for every predicate, 'is F (P)' there is a collection of things that are F. Two examples are: 'is a prime number' determines the collection of numbers {2, 3, 5, 7, 11...} and 'is a collection' determines the collection of all collections. In 1903 Russell showed that (P) is self-contradictory with the following argument. Consider the predicate 'is not a member of itself'. With (P) there is a collection – call it R – of collections that are not members of themselves. Is R a member of itself? If it is then it isn't, and if it isn't then it is. A contradiction. This was a devastating blow to Frege and to logicism.

## RELATED PHILOSOPHIES

see also  
ARISTOTLE'S SYLLOGISMS  
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## 3-SECOND BIOGRAPHY

BERTRAND RUSSELL  
1872–1970

## 30-SECOND TEXT

Barry Loewer

*He who shaves the  
shavers, shaves the  
least or the most?  
Either way, did anyone  
think about growing  
a beard?*



# ARISTOTLE

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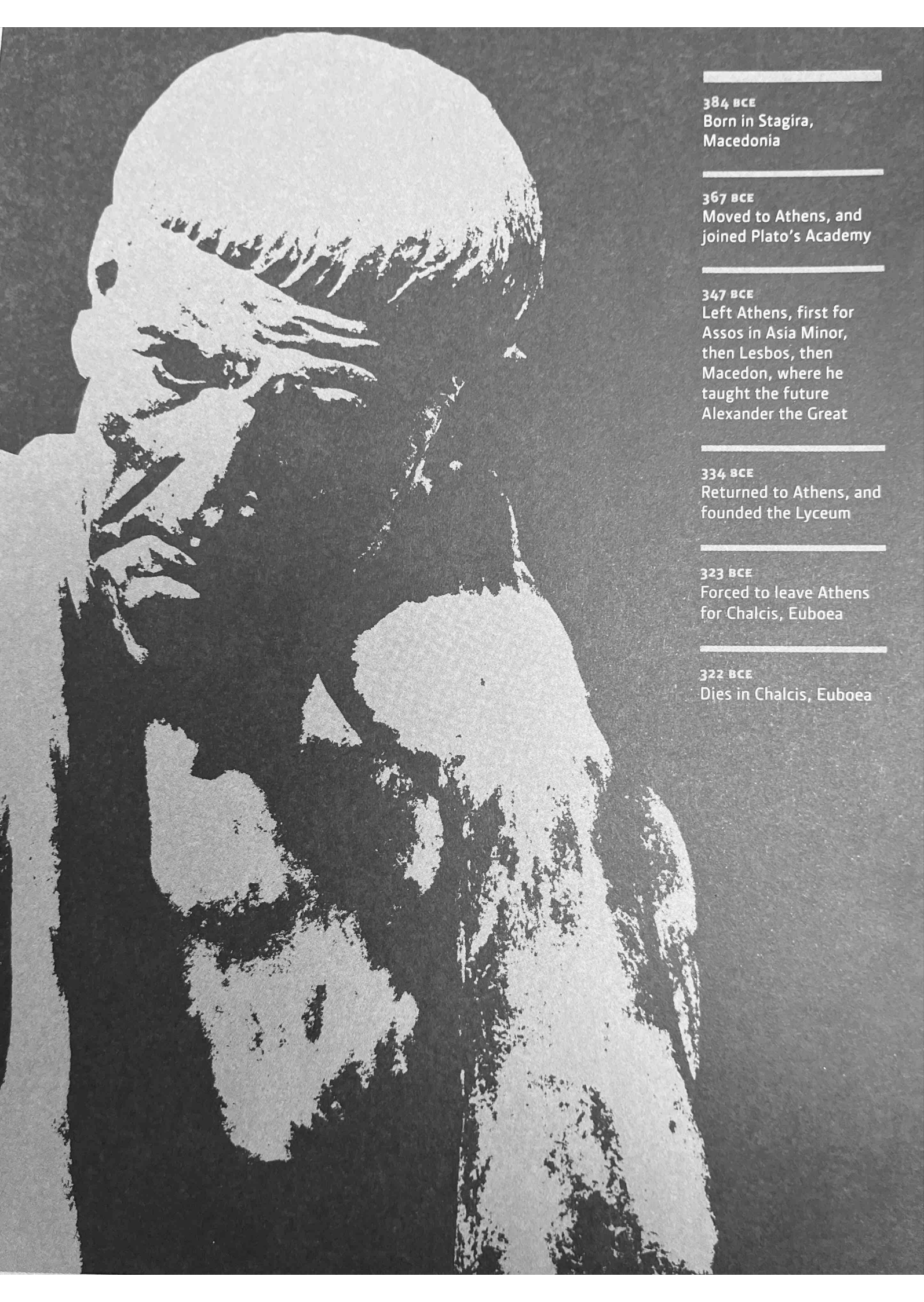
It would be hard to exaggerate the importance of Aristotle to the history of philosophy. As well as formalizing the rules of deduction, he undertook groundbreaking work in the fields of ethics, politics, metaphysics, biology, physics, psychology, aesthetics, poetry, rhetoric, cosmology, mathematics and the philosophy of mind.

Aristotle was born in 384 BCE, in the Macedonian city of Stagira, now in northern Greece. He was the son of Nicomachus, a physician to the court of the king of Macedon, who sent him to Athens in 367 BCE, where he joined Plato's Academy, remaining there for 20 years, first as a student and later as a teacher. After Plato's death, Aristotle left Athens, eventually ending up in Macedon, where he tutored the future Alexander the Great. He then returned to Athens, and founded his own school, the Lyceum or Peripatetic school (likely so called because he taught while strolling along the covered walkways

of the Lyceum). Aristotle remained in Athens until he ran into trouble in 323 BCE as anti-Macedonian sentiment swept the city, and charges of 'impiety' were laid against him. Insisting that he would not allow the Athenians to 'sin twice against philosophy', he left Athens for the city of Chalcis, where he died the following year of a digestive illness.

Unfortunately, we know less about the circumstances in which Aristotle produced his great works than we do about his life. It is likely that most of his surviving treatises were not intended for publication, but rather were assembled and edited from lecture notes by his successors. This in part explains why they are hard to read – full of technical language, detailed discussion, inconsistencies and *lacunae*. His work, nevertheless, remains one of the crowning achievements of the classical world, and probably unparalleled in its importance for the development of the discipline of philosophy.





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**384 BCE**  
Born in Stagira,  
Macedonia

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**367 BCE**  
Moved to Athens, and  
joined Plato's Academy

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**347 BCE**  
Left Athens, first for  
Assos in Asia Minor,  
then Lesbos, then  
Macedon, where he  
taught the future  
Alexander the Great

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**334 BCE**  
Returned to Athens, and  
founded the Lyceum

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**323 BCE**  
Forced to leave Athens  
for Chalcis, Euboea

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**322 BCE**  
Dies in Chalcis, Euboea

# RUSSELL'S THEORY OF DESCRIPTIONS

the 30-second philosophy

## 3-SECOND THRASH

The logical form of the statement: 'The present king of France is bald' is given by 'There is one and only one king of France, and he is bald'.

## 3-MINUTE THOUGHT

Underlying Russell's theory is the idea that a sentence has a 'logical form' that makes its meaning and its logic easily understood. This idea was very influential on subsequent philosophers and linguists, including Ludwig Wittgenstein and Noam Chomsky.

Bertrand Russell claimed that the reference of an expression is its meaning. At first he thought that the meaning of a definite description, for example 'the present king of France', was some particular object, in this case a particular king. But at that time France had no king, so Russell thought that the king must exist in some way, even though he couldn't be found in our world. Soon enough, Russell came to think that this was too much ontology to swallow and proposed his theory of descriptions to avoid this consequence, while holding onto the idea that reference is meaning. His idea is that 'the present king of France' doesn't have a meaning on its own, but any sentence in which this phrase occurs can be translated into a sentence in which the phrase doesn't occur. 'The present king of France is bald' is translated into 'There is one and only one present king of France, and he is bald'. If this is correct, then the original sentence with the definite description is false. Russell said that the second sentence revealed the logical form of the first sentence. Since the phrase 'the present king of France' doesn't occur in this sentence there is no need for a particular king to exist for the sentence to have meaning.

## RELATED PHILOSOPHIES

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FREGE'S PUZZLE  
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WITTGENSTEIN'S PICTURE  
THEORY OF LANGUAGE  
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## 3-SECOND BIOGRAPHIES

BERTRAND RUSSELL  
1872-1970

LUDWIG WITTGENSTEIN  
1889-1951

NOAM CHOMSKY  
1928-

## 30-SECOND TEXT

Barry Loewer

*Whatever Bertrand Russell says, this is definitely not the present king of France. He just wears the crown to cover his bald patch.*



# FREGE'S PUZZLE

the 30-second philosophy

## 3-SECOND THRASH

If 'Hesperus' and 'Phosphorus' are just different names for the same thing – the planet Venus – how can it be that 'Hesperus is Phosphorus' and 'Hesperus is Hesperus' differ in meaning?

## 3-MINUTE THOUGHT

Many philosophers find the notion of sense obscure. The logician Saul Kripke argued that proper names do not have senses at all. In his view the reference of a proper name is not determined by a sense but by a chain of uses of the name that begins with an act of naming. For example, you may use the name 'Thales' to refer to a certain pre-Socratic philosopher even though you don't know anything about him, as long as you acquired the name from someone who used it to refer to Thales.

In his early writings on language, the great logician Gottlob Frege held that the meaning of a name is its reference. For example, the meaning of the name 'Mont Blanc' is the mountain itself. But, in later writings, Frege argued that two names may have the same reference, yet differ in meaning. He reasoned that if the meaning of a name is just its reference, and two names have the same reference, then it should make no difference to the meaning of a sentence which name occurs in it. Since 'Hesperus' and 'Phosphorus' are both names of the planet Venus, (1) 'Hesperus is Phosphorus' and (2) 'Hesperus is Hesperus' should have the same meaning. But Frege observed that they do differ in meaning, since (1) expresses a significant astronomical discovery, while (2) is a triviality. The explanation of why they differ in meaning is Frege's Puzzle. Frege's solution is that the meaning of a name is not only its reference, but also its sense. The sense of a name is a condition that picks out the individual (if there is one) that satisfies that condition as the name's reference. Frege says that 'Hesperus' and 'Phosphorus' have different senses that pick out the same reference. This, he says, explains how (1) can be informative, while (2) is a triviality. Much twentieth-century philosophy of language involves a discussion of Frege's notion of sense.

## RELATED PHILOSOPHIES

see also  
RUSSELL'S PARADOX & FREGE'S  
LOGICISM  
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RUSSELL'S THEORY OF  
DESCRIPTIONS  
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## 3-SECOND BIOGRAPHY

GOTTLIB FREGE  
1848–1925

## 30-SECOND TEXT

Barry Loewer

*You say Phosphorus,  
I say Hesperus. Let's  
call the whole thing off –  
and just call it Venus.*

# GÖDEL'S THEOREM

## the 30-second philosophy

### 3-SECOND THRASH

For any (sufficiently strong) mathematical theory, there are true statements that cannot be proved in that theory.

### 3-MINUTE THOUGHT

Some philosophers, and the physicist Roger Penrose, have claimed that Gödel's theorem shows that our minds do not work like computers. Following a programme is analogous to proving a theorem. Gödel showed that, for any axiom system, the statement that the system is consistent cannot be proved by the system itself. So, if our minds operated like a computer following a programme, we could not recognize that we are consistent. But we seem able to recognize our own consistency, therefore our minds do not work like computers.

Gödel's theorem is the most profound result in mathematical logic. It is thought to have important philosophical consequences for the limits of knowledge and the nature of mind. In the system of modern logic, it is possible to express arithmetical statements, for example, 'For any pair of numbers  $n$  and  $m$ ,  $n + m = m + n$ '. It is also possible to write down axioms (called 'Peano's axioms'), from which one can prove many mathematical truths. The question arose of whether one can prove from these axioms all arithmetical truths, without proving any false statements. Kurt Gödel answered this question negatively. First, he discovered a coding whereby arithmetical statements also have an interpretation in which they are about themselves and what can be proved from various axioms. He then found an arithmetical statement (K) that says under the coding '(K) is not provable'. He reasoned that if (K) is provable then the axioms prove a false statement. But if (K) is not provable then it is true, and there is a truth that the axioms don't prove. Not only are there arithmetical truths that cannot be proved from Peano's axioms, but also any true axioms will leave out some truths as unprovable. This is known as 'Gödel's incompleteness theorem'. It seems to establish a limit on what mathematicians can know.

### RELATED PHILOSOPHIES

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EPIMENIDES' LIAR PARADOX  
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### 3-SECOND BIOGRAPHIES

KURT GÖDEL  
1906–78

ROGER PENROSE  
1931–

### 30-SECOND TEXT

Barry Loewer

*Even by replacing his brain with a computer, Kurt was unable to figure out those unknowable truths.*



# EPIMENIDES' LIAR PARADOX

the 30-second philosophy

**3-SECOND THRASH**  
This sentence is false.

**3-MINUTE THOUGHT**  
The concept of truth is too important to philosophy and scientific thinking to overlook, so there have been many attempts to solve the liar paradox. Tarski's idea gives up on there being a single concept of truth applicable to all languages. Other philosophers have responded by restricting inferences from S to 'S' is true, and some have even developed logics in which certain contradictions are acceptable.

Epimenides was a sixth-century BCE Cretan philosopher who is reputed to have said, 'All Cretans are liars'. If his utterance is true then he is lying, and what he said is false. This is an ancient version of what has come to be known as 'the liar paradox'. A contemporary version is based on '1. Sentence 1 is not true'. If sentence 1 is true then it is not true, and if it is not true then it is true. The paradox arises because it seems to be part of the meaning of 'is true' that, where S can be any sentence, one can validly infer 'S' is true from S and also validly infer S from 'S' is true. From 1 we can infer both that S is true and that S is not true. A paradox. The most famous response to the liar paradox came from the logician Alfred Tarski, who distinguished a language (L) from a meta-language (ML), in which one can refer to sentences of L. It is possible to define 'is true in L' in ML without paradox.

## RELATED PHILOSOPHIES

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GÖDEL'S THEOREM  
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## 3-SECOND BIOGRAPHIES

EPIMENIDES  
sixth century BCE  
ALFRED TARSKI  
1901–83

## 30-SECOND TEXT

Barry Loewer

*Which one of you  
said, 'The truth is rarely  
pure and never simple'?  
Whoever it was, you're  
a liar.*

# EUBULIDES' HEAP

## the 30-second philosophy

### 3-SECOND THRASH

Why you can never make a mountain out of a molehill.

### 3-MINUTE THOUGHT

What might this paradox show? That concepts such as fat and thin are vague, so it is a mistake to ever treat them as though there were factual questions to which they definitely apply? Or is it that, counter-intuitively, there is a firm boundary between fat and thin, a heap and a small pile, and that, if you step across it by one grain or 1 gram, the correct description changes?

### Weighing in at 100 kilograms

(220.462 pounds), Harry is a fat man. He won't stop being fat if his weight drops to 99.999 kilos (220.460 pounds). So that means any man who weighs the same as Harry must also be fat: a fraction of a gram, or an ounce, can never make the difference between being fat or thin. But, if that's true, then someone who weighs 99.998 kilos (220.457 pounds) is also fat, and so is someone who weighs 99.997 kilos (220.455 pounds) and so on. You'll still be claiming that a fraction of a gram, or an ounce, can't make the difference between fat and thin when you're comparing the person who weighs 40 kilos (88.184 pounds) to one who weighs 39.999 kilos (88.182 pounds). But this is absurd: someone who weighs 40 kilos (88.184 pounds) could never be described as fat. Hence the paradox: a series of apparently logically watertight steps leads us to a conclusion that is manifestly false. But neither the logic nor the observation have any evident flaws. This is a version of Eubulides paradox of the heap, where a similar argument showed that a heap would still be a heap when it contained only one grain of sand, just as long as the grains were removed one by one.

### 3-SECOND BIOGRAPHY

EUBULIDES  
fourth century BCE

### 30-SECOND TEXT

Julian Baggini

*The journey from being a thin man to being a fat man and back again begins with a single gram.*